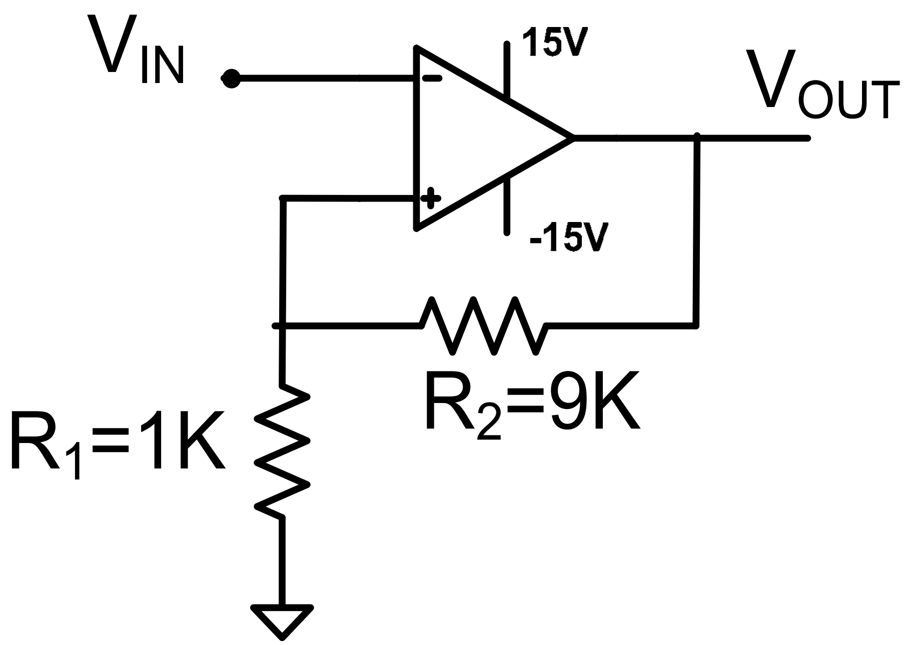
EE 230
Lecture 22

Nonlinear Op Amp Applications

Quiz 15

Give a mathematical expression for the transfer characteristics of the following circuit.



And the number is ?

1 3 8

5 4

2 6

9 7

And the number is ?

1 3 8

5 4

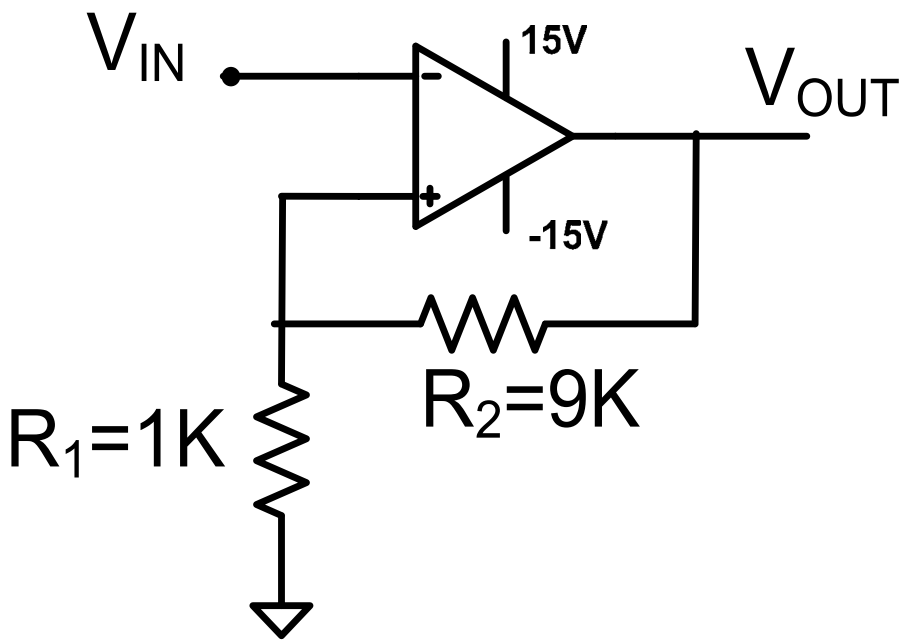
7

2 6

9 7

Quiz 15

Give a mathematical expression for the transfer characteristics of the following circuit.

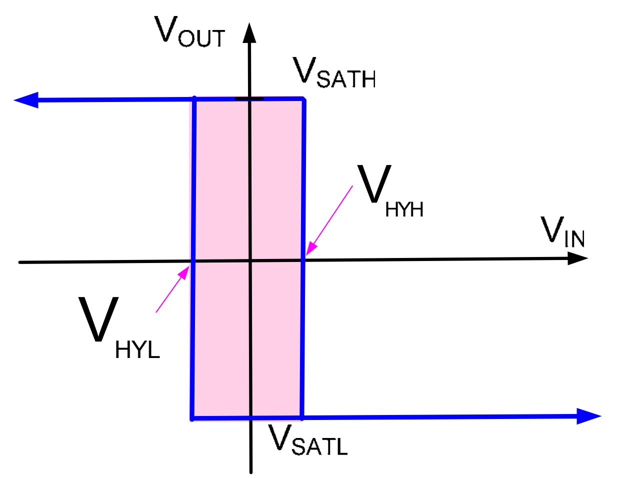


Solution:

Observe this is an Inverting Comparator with Hysteresis

Define $\theta = \frac{R_1}{R_1 + R_2}$

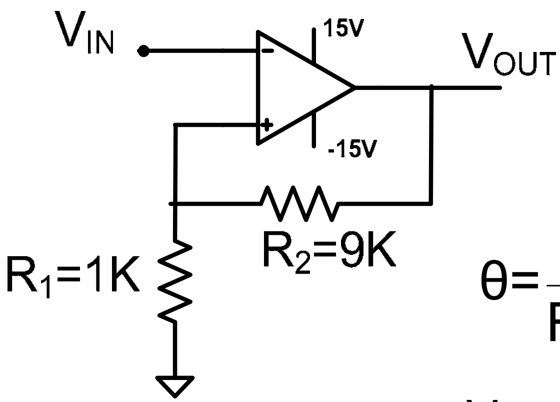
Observe $V_{DD} = V_{SS} = 15V$



$$V_{OUT} = \begin{cases} V_{SATH} & V_{IN} < V_{HYH} \\ V_{SATL} & V_{IIN} > V_{HYL} \end{cases}$$

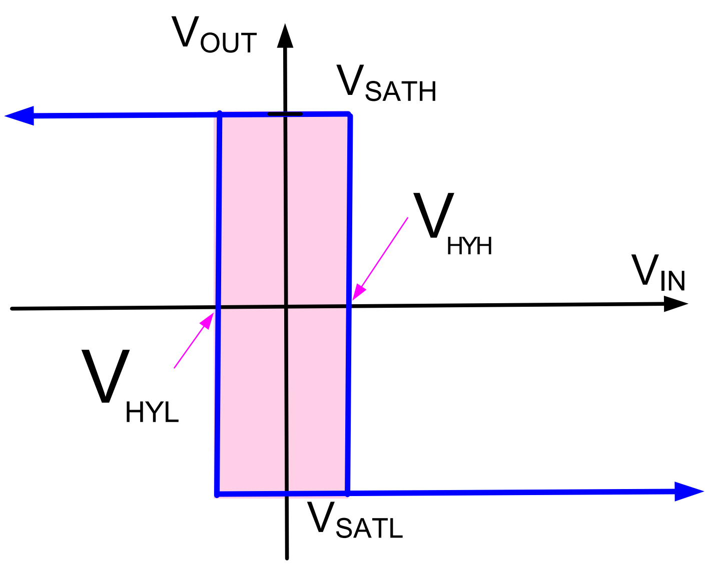
Quiz 15

Give a mathematical expression for the transfer characteristics of the following circuit.



$$\theta=\frac{R_1}{R_1+R_2}$$

$$V_{DD}=V_{SS}=15V$$



Solution:

$$V_{SATH}=V_{DD} \qquad V_{SATL}=-V_{DD}$$

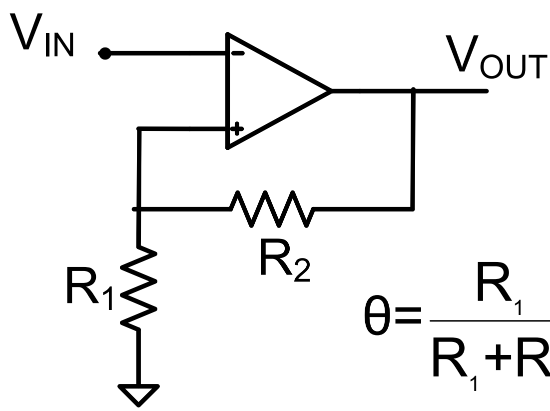
$$V_{HYH}=\theta V_{SATH}=\theta V_{DD}$$

$$V_{HYL}=\theta V_{SATL}=-\theta V_{DD}$$

$$\theta=\frac{1K}{1K+9K}=0.1$$

$$V_{OUT}=\begin{cases}15V & V_{IN}<1.5V \\ -15V & V_{IIN}>-1.5V\end{cases}$$

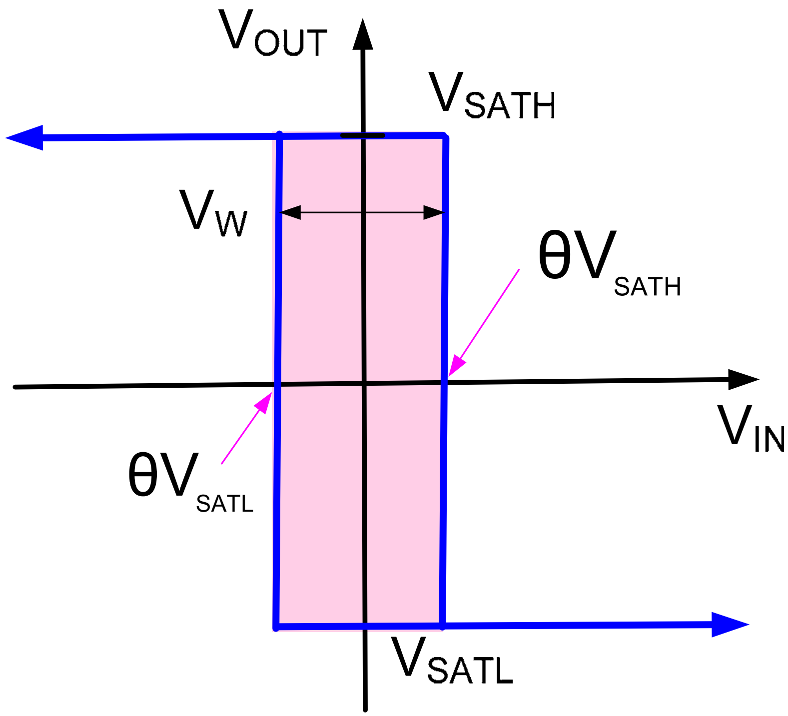
Review from Last Time:



$$\theta = \frac{R_1}{R_1 + R_2}$$

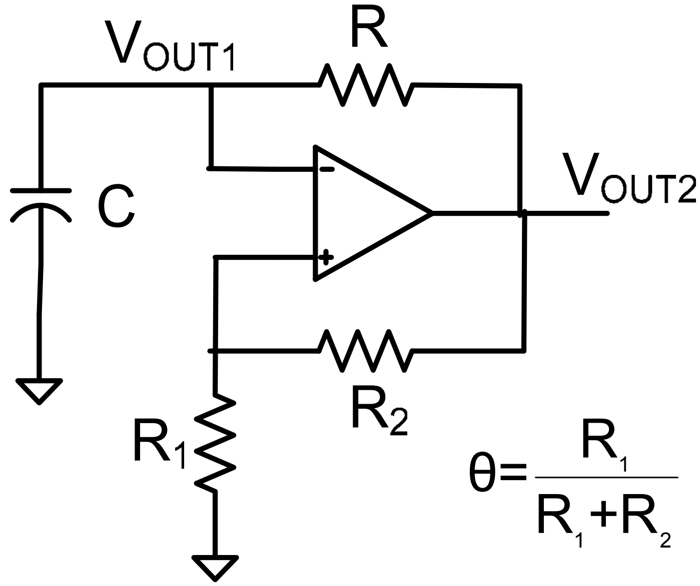
Inverting Comparator with Hysteresis

$$V_{OUT} = \begin{cases} V_{SATH} & V_{IN} < V_{HYH} \\ V_{SATL} & V_{IIN} > V_{HYL} \end{cases}$$



$$V_W = \theta\left(V_{SATH} - V_{SATL}\right)$$

Review from Last Time: Waveform Generator



$$\theta = \frac{R_1}{R_1 + R_2}$$

$$V_{SATH} \cong V_{DD}$$

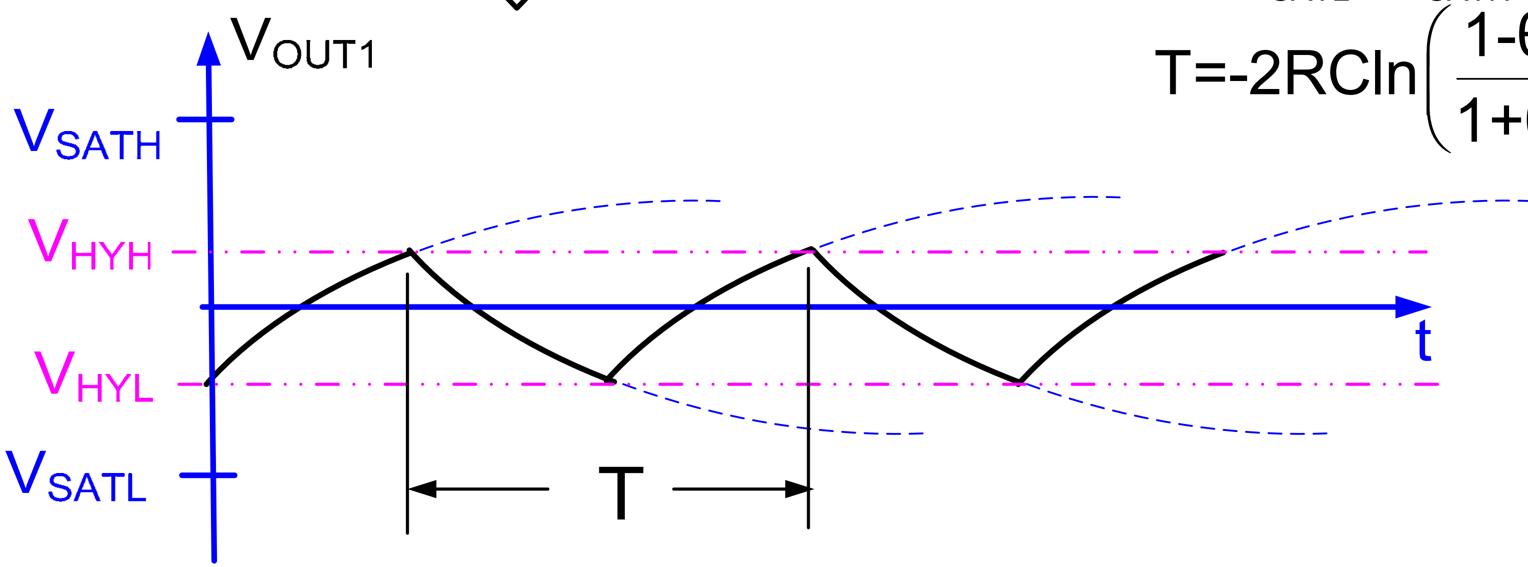
$$V_{SATL} \cong V_{SS}$$

$$V_{HYH} = \theta V_{SATH}$$

$$V_{HYL} = \theta V_{SATL}$$

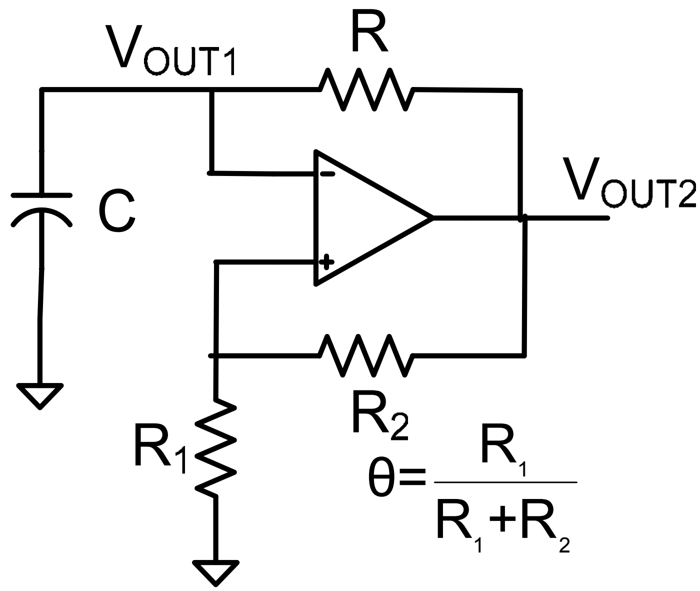
for $V_{SATL} = -V_{SATH}$

$$T = -2RC\ln\left(\frac{1-\theta}{1+\theta}\right)$$



Review from Last Time:

Waveform Generator



$$\theta = \frac{R_1}{R_1 + R_2}$$

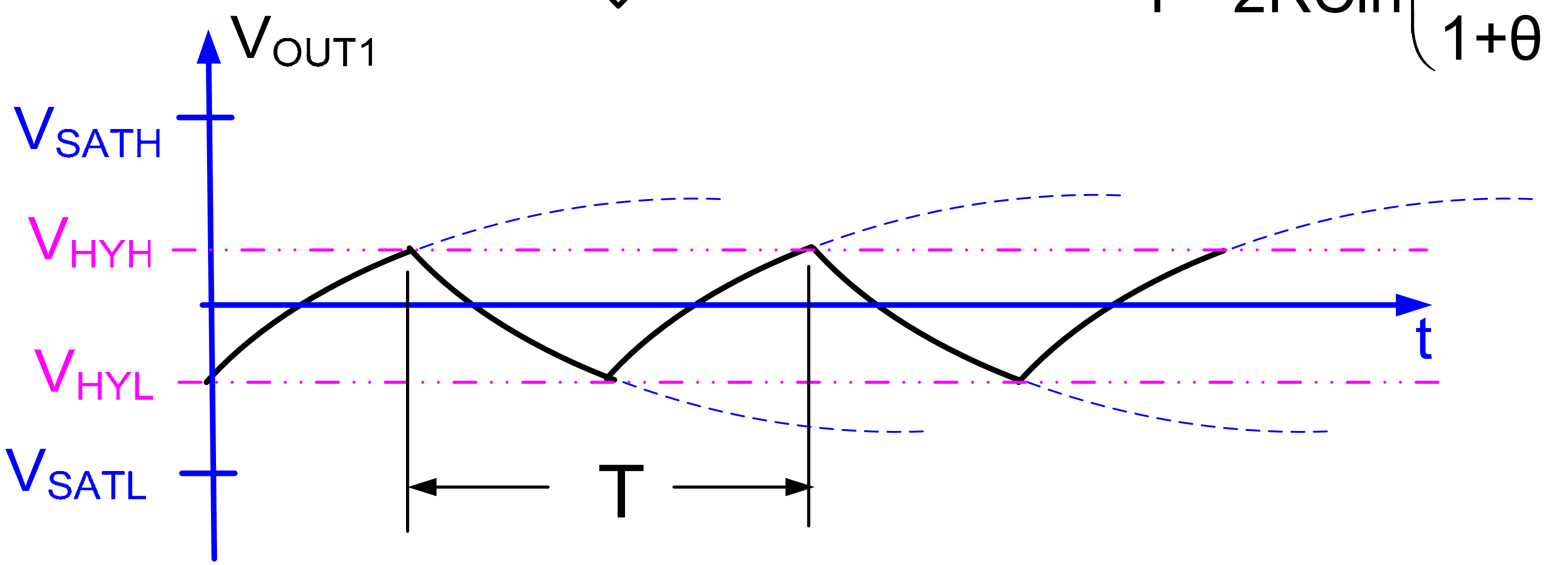
$$V_{SATH} \cong V_{DD}$$

$$V_{SATL} \cong V_{SS}$$

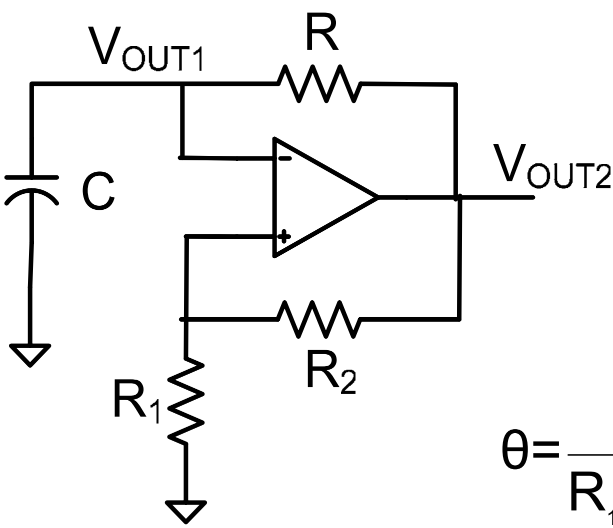
$$V_{HYH} = \theta V_{SATH}$$

$$V_{HYL} = \theta V_{SATL}$$

$$T = -2RC\ln\left(\frac{1-\theta}{1+\theta}\right)$$



Review from Last Time:



$$V_{SATH} \cong V_{DD}$$

$$V_{SATL} \cong V_{SS}$$

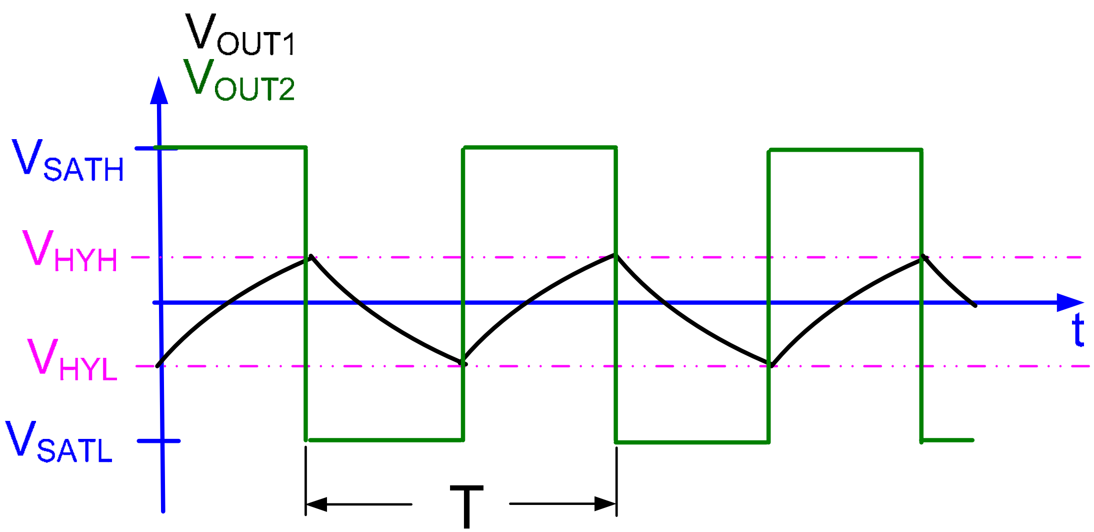
$$V_{HYH} = \theta V_{SATH}$$

$$V_{HYL} = \theta V_{SATL}$$

$$\theta = \frac{R_1}{R_1 + R_2}$$

for $V_{SATL} = -V_{SATH}$

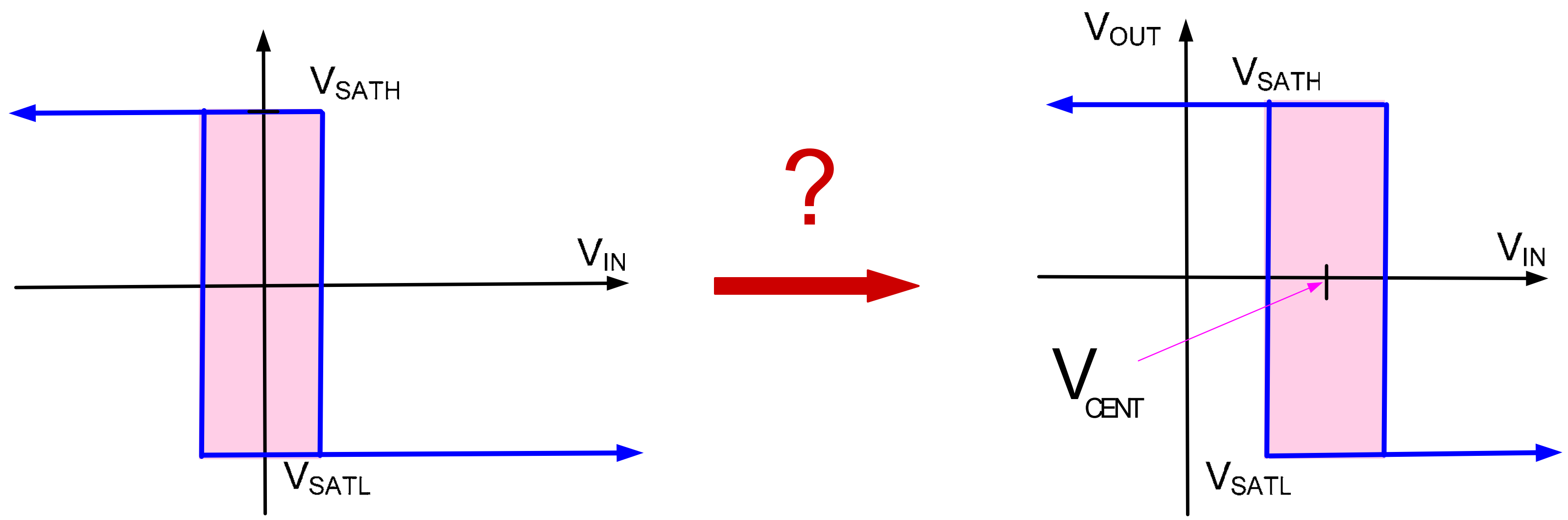
$$f = \frac{1}{2RC} \frac{-1}{\ln\left(\frac{1-\theta}{1+\theta}\right)}$$



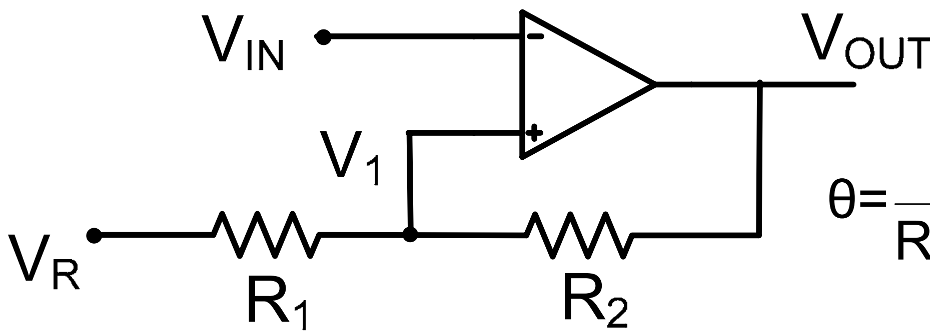
Square and distorted triangular output waveforms

Slope of square wave is determined by SR of Op Amp

Movement of Hysteresis Loop



Movement of Hysteresis Loop



$$\theta = \frac{R_1}{R_1 + R_2} \qquad 1-\theta = \frac{R_2}{R_1 + R_2}$$

Consider adding a dc voltage V_R

Edges of hysteresis loop determined by condition where $V^+ = V^-$

For this circuit that is where

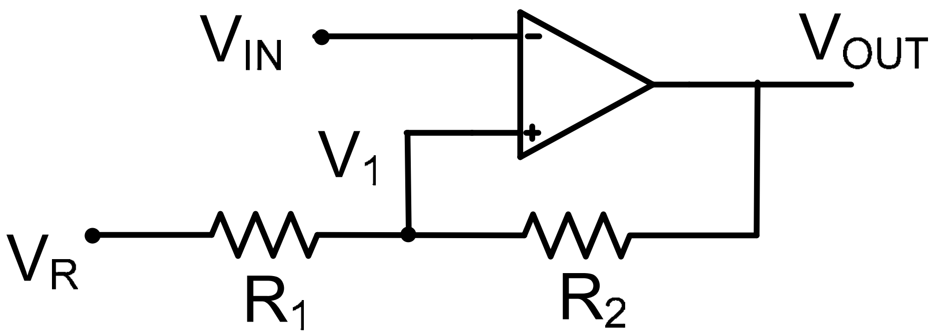
$$V_1 = V_{IN}$$

It follows from the 2-input voltage divider equation that

$$V_1 = \theta V_{OUT} + (1-\theta) V_R$$

Substituting the second equation into the first, the edges of the hysteresis loop can be obtained by solving for the two possible values of V_{OUT}, V_{SATH} and V_{SATL}

Movement of Hysteresis Loop

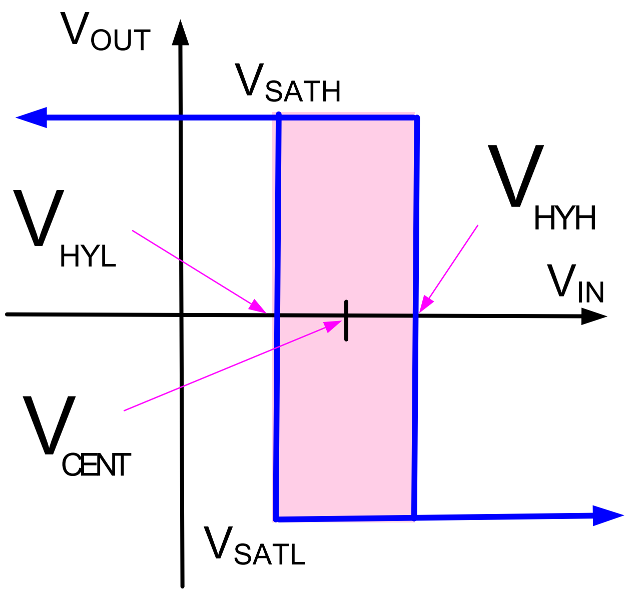


Shifted Inverting Comparator with Hysteresis

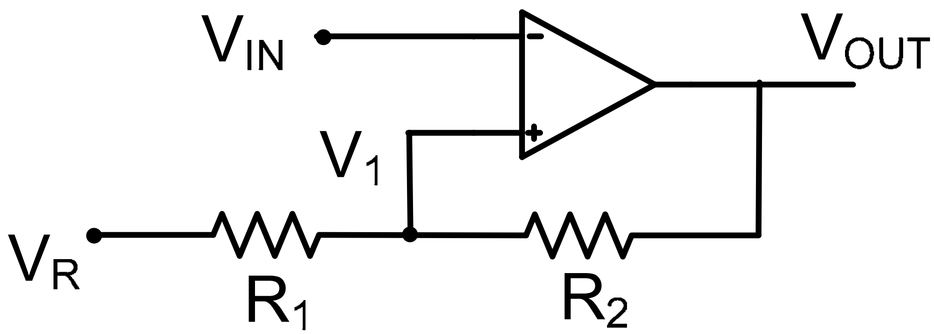
$$\theta = \frac{R_1}{R_1 + R_2} \qquad 1-\theta = \frac{R_2}{R_1 + R_2}$$

$$V_{SATH} \cong V_{DD} \qquad V_{SATL} \cong V_{SS}$$

$$V_{IN} = \theta V_{OUT} + (1-\theta) V_R \rightarrow \begin{cases} V_{HYH} = \theta V_{SATH} + (1-\theta) V_R \\ V_{HYL} = \theta V_{SATL} + (1-\theta) V_R \end{cases}$$



Movement of Hysteresis Loop

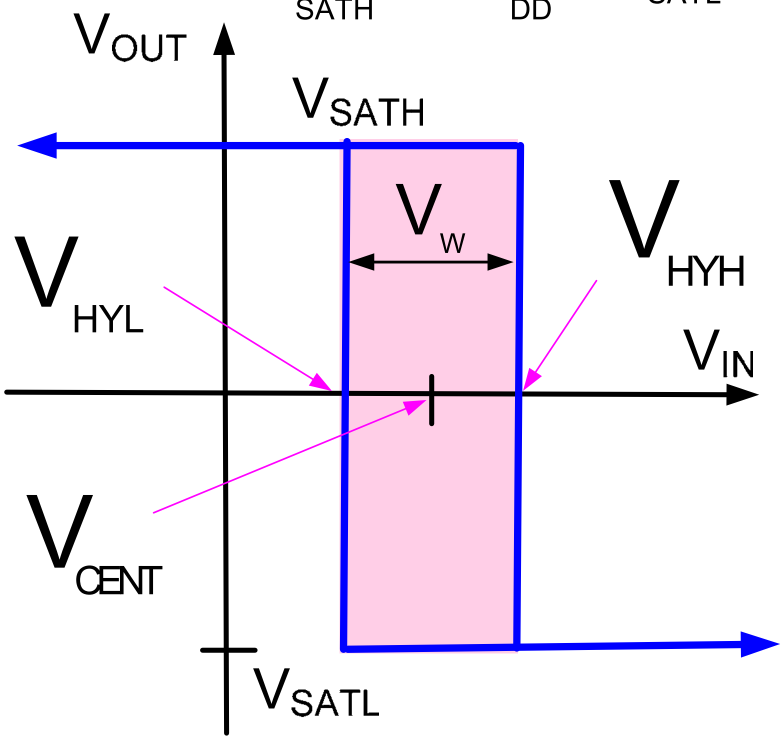


$$\theta = \frac{R_1}{R_1 + R_2} \qquad 1-\theta = \frac{R_2}{R_1 + R_2}$$

$$V_{HYH} = \theta V_{SATH} + (1-\theta) V_R$$

$$V_{HYL} = \theta V_{SATL} + (1-\theta) V_R$$

$$V_{SATH} \cong V_{DD} \qquad V_{SATL} \cong V_{SS}$$



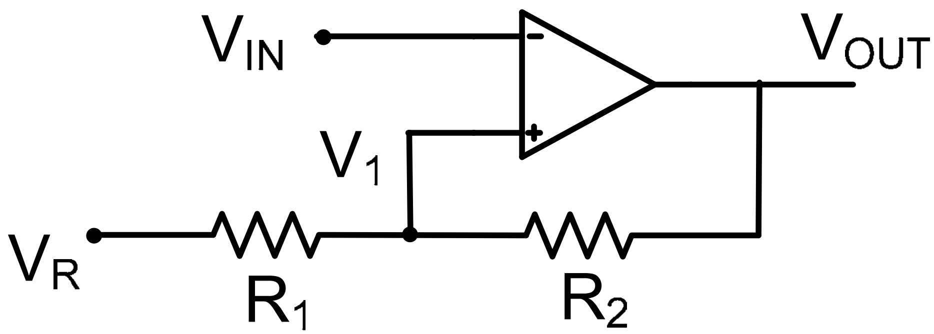
$$V_W = V_{HYH} - V_{HYL}$$

$$V_W = \theta\left(V_{SATH} - V_{SATL}\right)$$

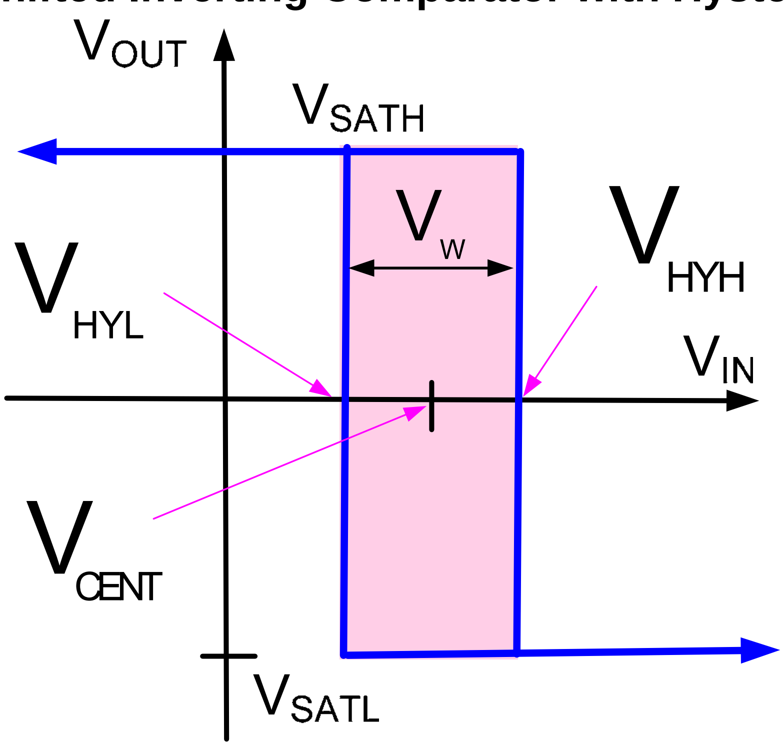
$$V_{CENT} = \frac{V_{HYH} + V_{HYL}}{2}$$

$$V_{CENT} = \left(\frac{\theta\left(V_{SATH} + V_{SATL}\right)}{2}\right) + (1-\theta) V_R$$

Movement of Hysteresis Loop



Shifted Inverting Comparator with Hysteresis



$$\theta = \frac{R_1}{R_1 + R_2}$$

$$V_W = \theta\left(V_{SATH} - V_{SATL}\right)$$

$$V_{CENT} = \left(\frac{\theta\left(V_{SATH} + V_{SATL}\right)}{2}\right) + \left(1 - \theta\right)V_R$$

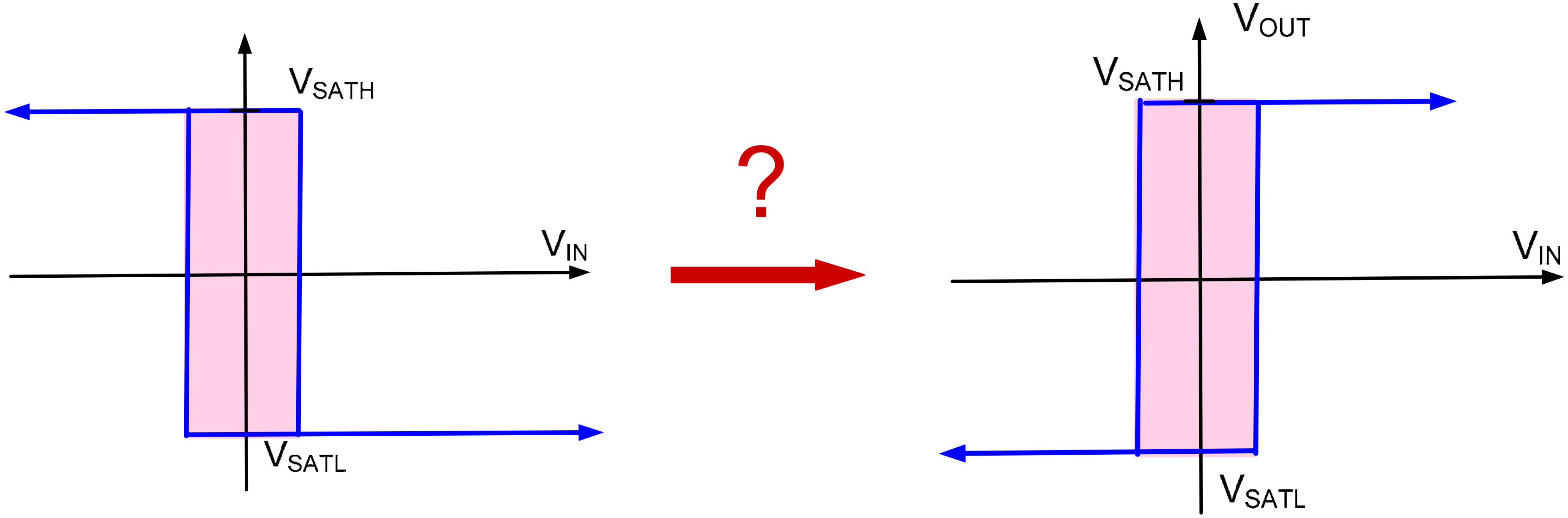
If $V_{SATH} = V_{DD}$, $V_{STAL} = V_{SS} = -V_{DD}$

$$V_W = 2\theta V_{DD}$$

$$V_{CENT} = \left(1 - \theta\right)V_R$$

Shift can be to left or right depending upon sign of V_R

Inversion of Hysteresis Loop

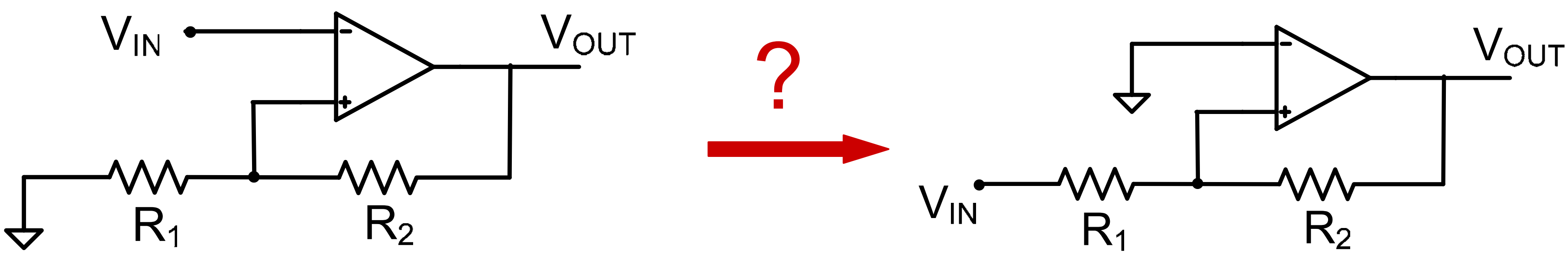


Inverting Comparator with Hysteresis **Noninverting Comparator with Hysteresis**

Strategies

- **Precede or follow inverting structure with an inverting amplifier**
- **Modify input location**

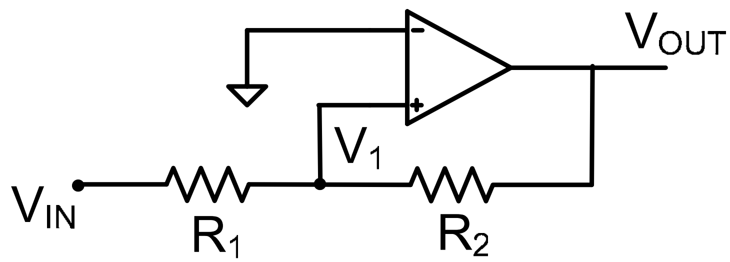
Inversion of Hysteresis Loop



Inverting Comparator with Hysteresis

Noninverting Comparator with Hysteresis ?

Inversion of Hysteresis Loop



$$\theta = \frac{R_1}{R_1 + R_2}$$

Noninverting Comparator with Hysteresis ?

Edges of hysteresis loop determined by condition where $V^+ = V^-$

For this circuit that is where

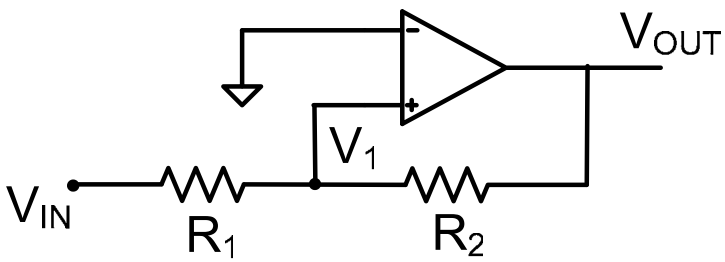
$$V_1 = 0$$

It follows from the 2-input voltage divider equation that

$$V_1 = \theta V_{OUT} + (1-\theta) V_{IN}$$

Substituting the second equation into the first, the edges of the hysteresis loop can be obtained by solving for the two possible values of V_{OUT}, V_{SATH} and V_{SATL}

Inversion of Hysteresis Loop



$$\theta = \frac{R_1}{R_1 + R_2}$$

$$V_{SATH} \cong V_{DD} \qquad V_{SATL} \cong V_{SS}$$

Noninverting Comparator with Hysteresis ?

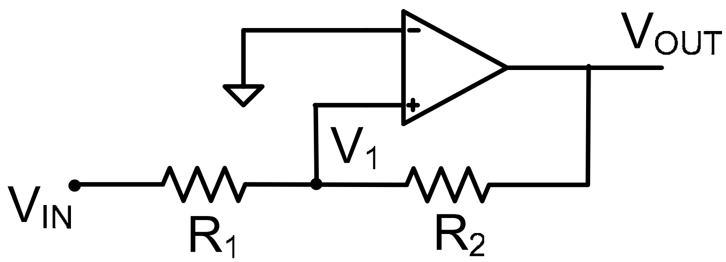
$$0 = \theta V_{OUT} + (1-\theta) V_{IN} \rightarrow \begin{cases} 0 = \theta V_{SATH} + (1-\theta) V_{HYL} \\ 0 = \theta V_{SATL} + (1-\theta) V_{HYH} \end{cases}$$

Solving, we obtain

$$V_{HYH} = \frac{\theta}{\theta - 1} V_{SATL}$$

$$V_{HYL} = \frac{\theta}{\theta - 1} V_{SATH}$$

Inversion of Hysteresis Loop



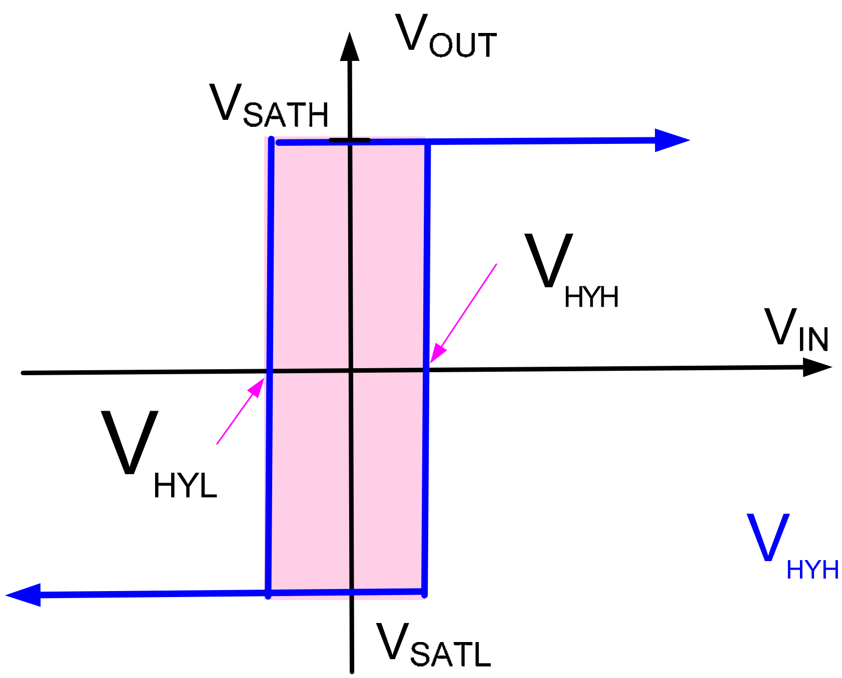
Noninverting Comparator with Hysteresis

$$\theta = \frac{R_1}{R_1 + R_2}$$

$$V_{SATH} \cong V_{DD} \qquad V_{SATL} \cong V_{SS}$$

$$V_{HYH} = \frac{\theta}{\theta - 1} V_{SATL}$$

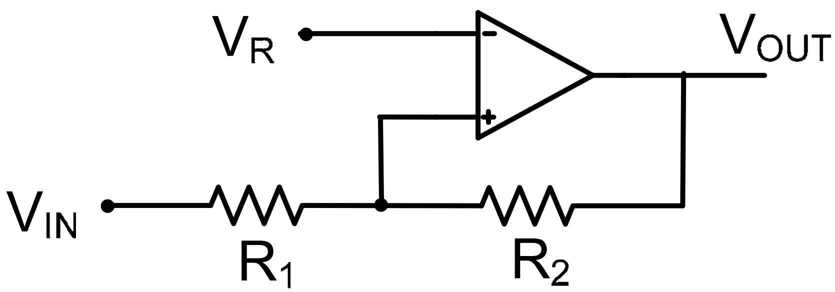
$$V_{HYL} = \frac{\theta}{\theta - 1} V_{SATH}$$



If $V_{SATH} = V_{DD}$, $V_{STAL} = V_{SS} = -V_{DD}$

$$V_{HYH} = \frac{\theta}{1 - \theta} V_{DD} \qquad V_{HYL} = \frac{-\theta}{1 - \theta} V_{DD}$$

Shifted Inverted Hysteresis Loop



Shifted Noninverting Comparator with Hysteresis

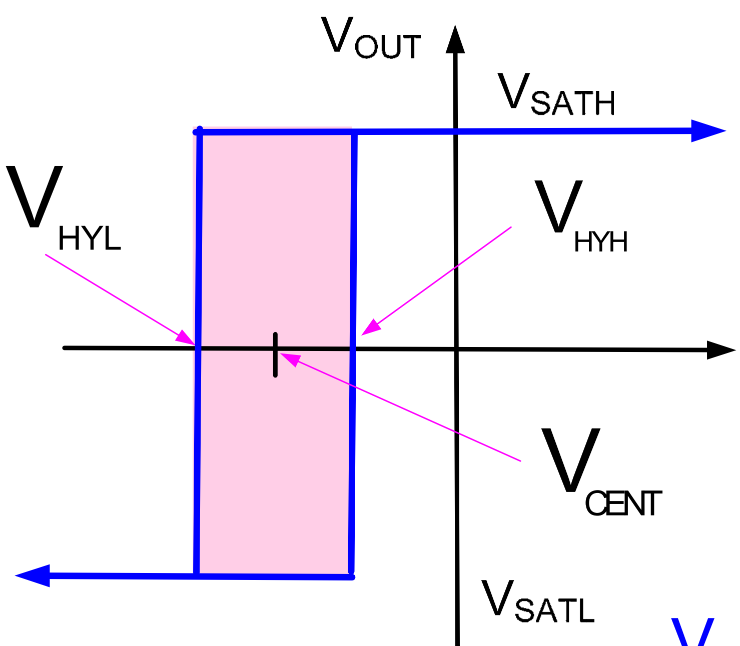
$$\theta = \frac{R_1}{R_1 + R_2}$$

$$V_{SATH} \cong V_{DD} \qquad V_{SATL} \cong V_{SS}$$

$$V_{HYH} = \frac{V_R}{1-\theta} + \frac{\theta}{\theta-1} V_{SATL}$$

$$V_{HYL} = \frac{V_R}{1-\theta} + \frac{\theta}{\theta-1} V_{SATH}$$

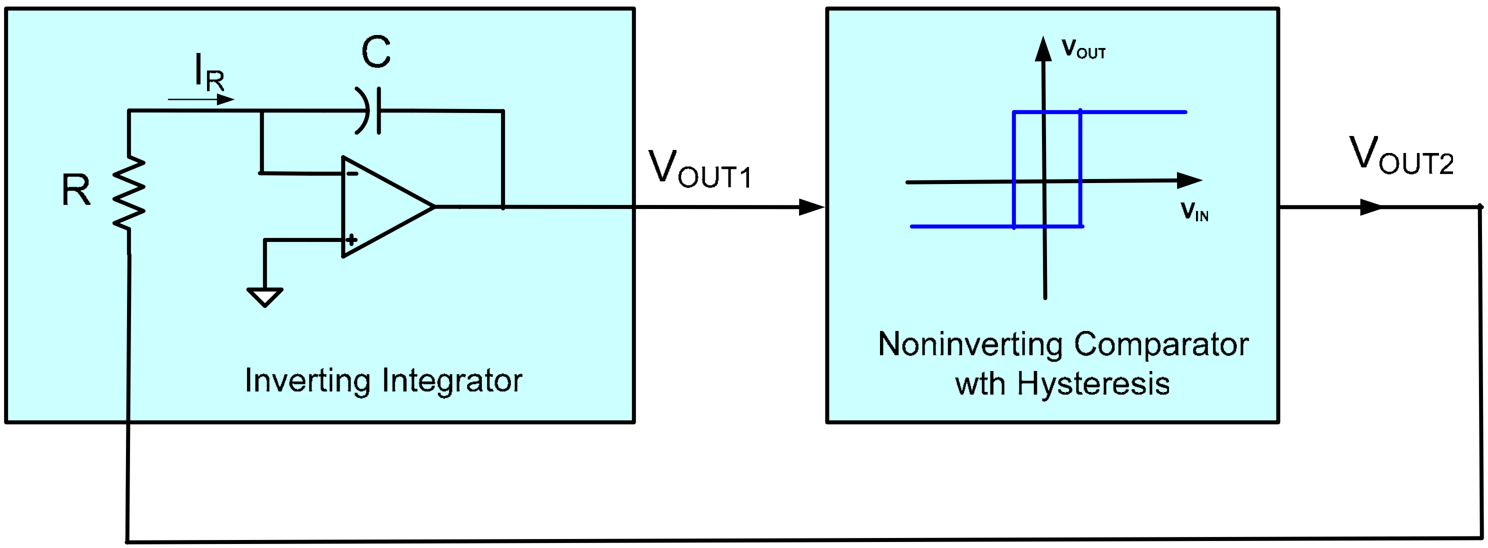
$$V_{CENT} = \frac{V_R}{1-\theta} + \frac{\theta}{\theta-1}\left(\frac{V_{SATH} - V_{SATL}}{2}\right)$$



If $V_{SATH}=V_{DD}$, $\quad V_{STAL}=V_{SS}=-V_{DD}$

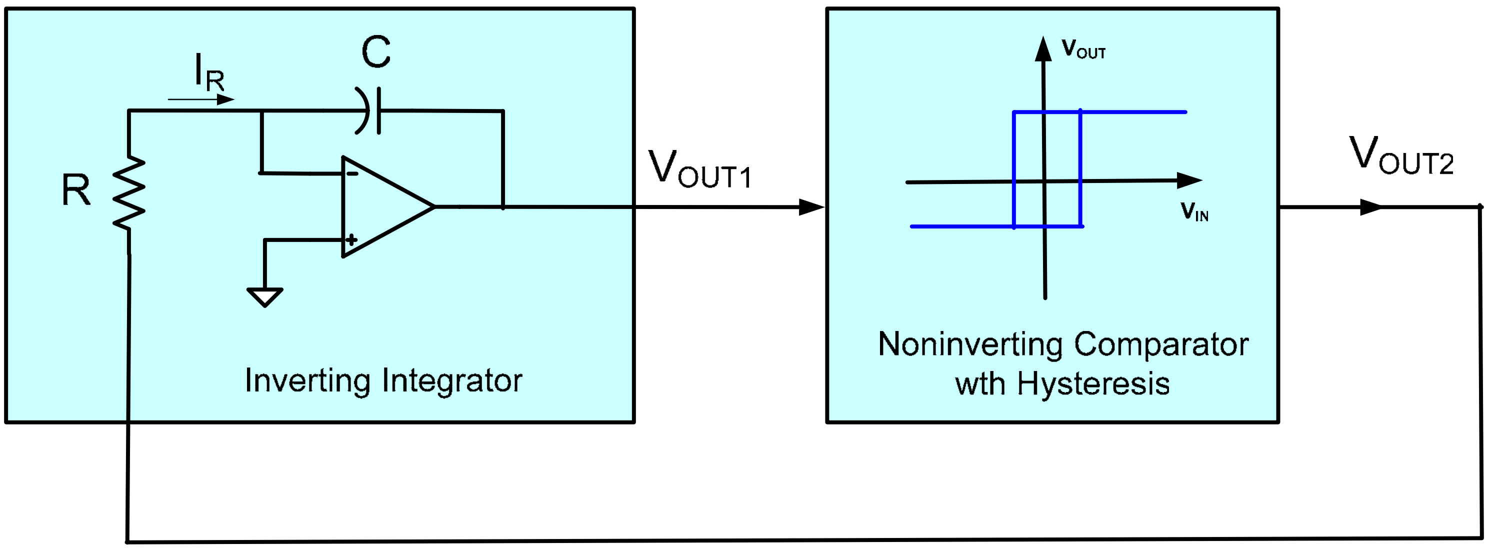
$$V_{HYH} = \frac{V_R}{1-\theta} + \frac{\theta}{1-\theta} V_{DD} \qquad V_{HYL} = \frac{V_R}{1-\theta} + \frac{-\theta}{1-\theta} V_{DD}$$

Waveform Generator with Linear Triangle Waveform



Goal: Determine how this circuit operates, the output waveforms, and the frequency of the output

Waveform Generator with Linear Triangle Waveform



Since the comparator will be in one of two states, the current in the resistor will be constant when $V_{OUT2}=V_{SATH}$ and will be constant when $V_{OUT2}=V_{SATL}$

Analysis strategy: Guess state of the V_{OUT2}, solve circuit, and show where valid

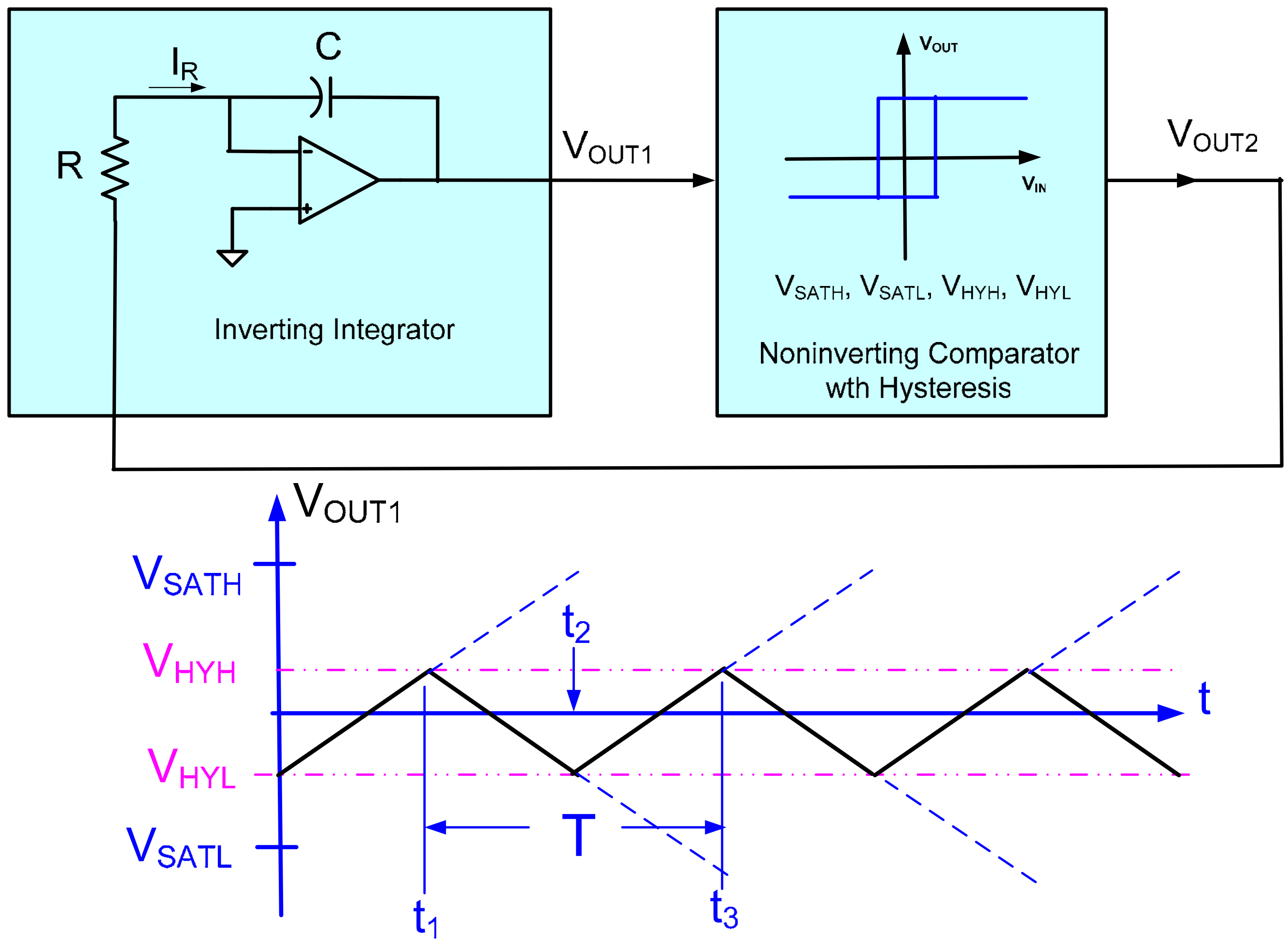
when $V_{OUT2}=V_{SATH}$, I_R will be positive and V_{OUT1} will be decreasing linearly

when $V_{OUT2}=V_{SATH}$, I_R will be positive and V_{OUT1} will be increasing linearly

Waveform Generator with Linear Triangle Waveform

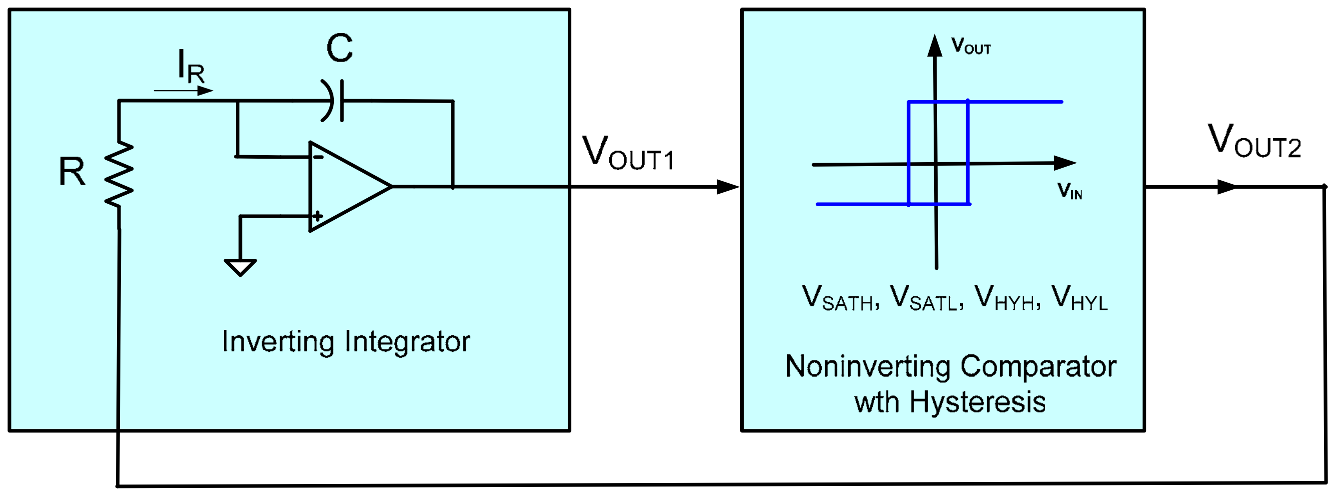
$V_{SATH} \cong V_{DD}$

$V_{SATL} \cong V_{SS}$



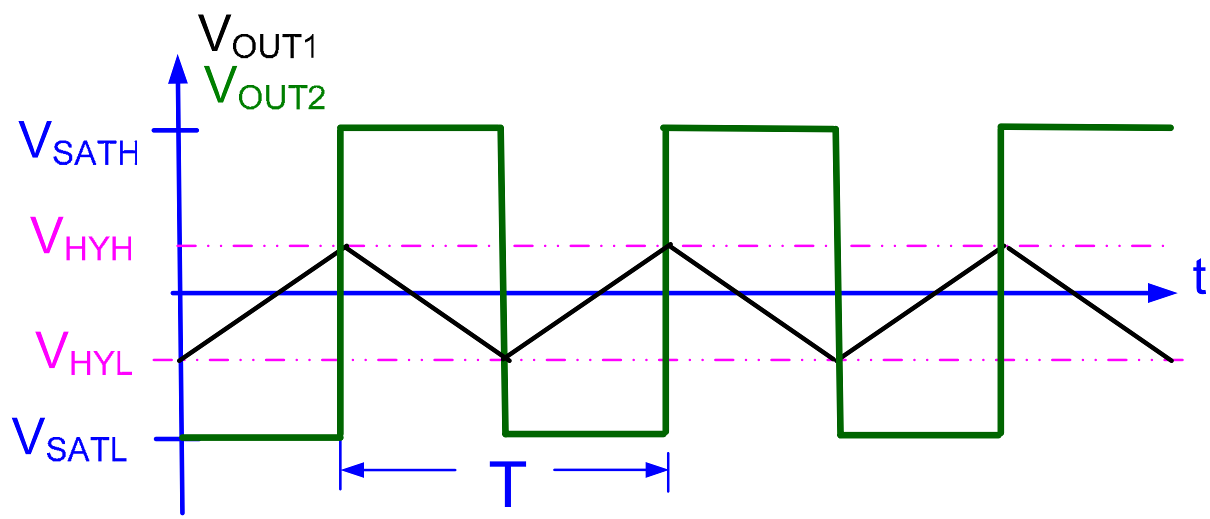
Observe $T = t_3 - t_1 = (t_2 - t_1) + (t_3 - t_2)$

Waveform Generator with Linear Triangle Waveform



$$V_{SATH} \cong V_{DD}$$

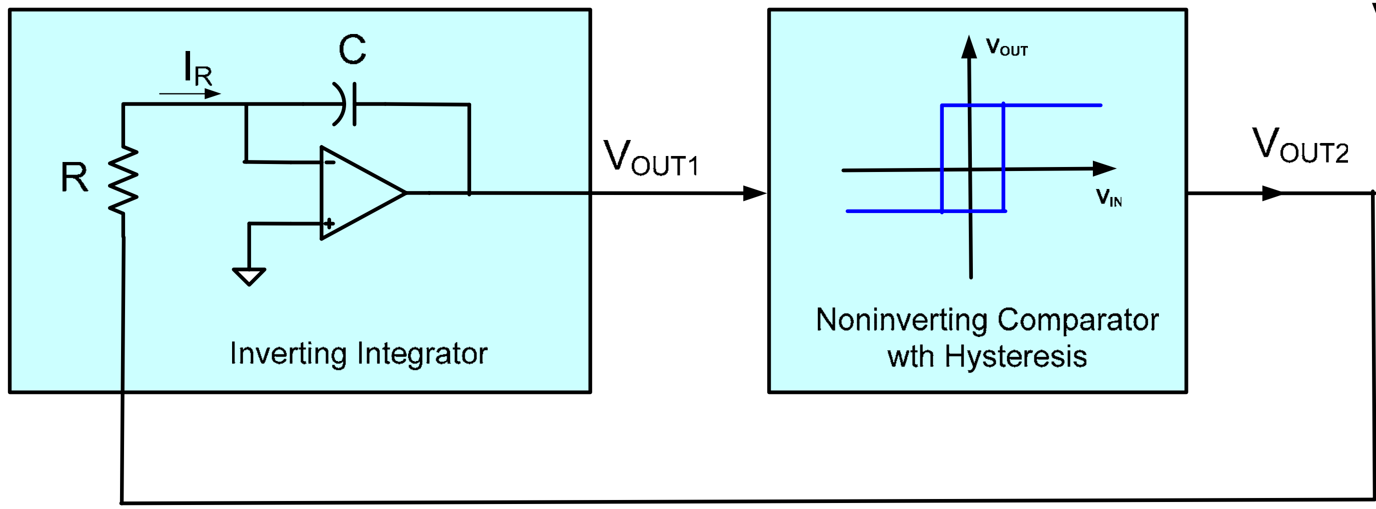
$$V_{SATL} \cong V_{SS}$$



Waveform Generator with Linear Triangle Waveform

$V_{SATH} \cong V_{DD}$

$V_{SATL} \cong V_{SS}$



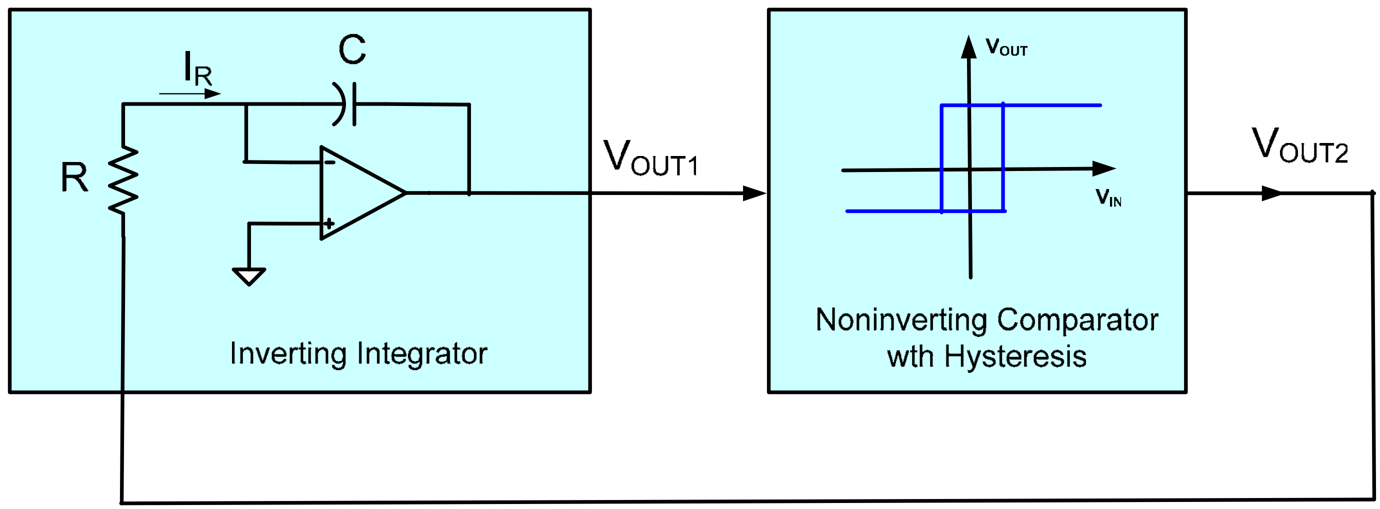
Guess $V_{OUT2}=V_{SATH}$ will obtain t_2-t_1

$$V_{OUT1} = -\frac{1}{RC}\int_{t_1}^{t} V_{SATH}\,d\tau + V_{OUT1}(t_1)$$

$$V_{OUT1}(t_1) = V_{HYH}$$

valid for $t_1 < t < t_2$

Waveform Generator with Linear Triangle Waveform



$$V_{SATH} \cong V_{DD}$$
$$V_{SATL} \cong V_{SS}$$

Guess $V_{OUT2}=V_{SATH}$ valid for $t_1 < t < t_2$

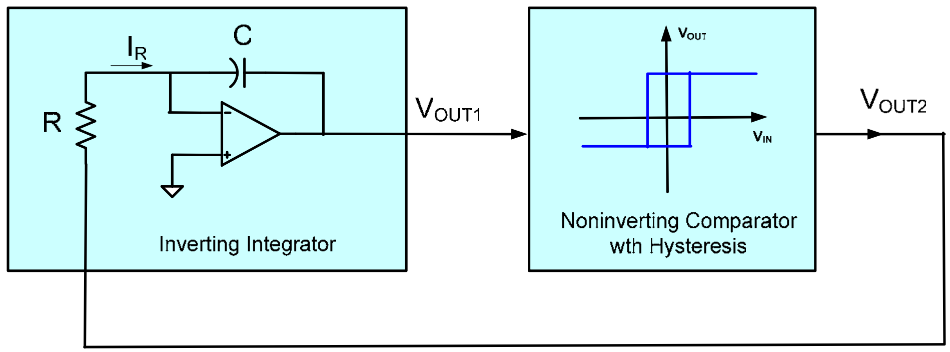
$$V_{OUT1} = -\frac{1}{RC}\int_{t_1}^{t} V_{SATH}\,d\tau + V_{OUT1}(t_1) \qquad V_{OUT1}(t_1) = V_{HYH}$$

at $t=t_2$, V_{OUT1} will become V_{SATL}

Substituting into integral expression for V_{OUT1} we obtain

$$V_{HYL} = -\frac{1}{RC}\int_{t_1}^{t} V_{SATH}\,d\tau + V_{HYH}$$

Waveform Generator with Linear Triangle Waveform



$$V_{SATH} \cong V_{DD}$$

$$V_{SATL} \cong V_{SS}$$

Guess $V_{OUT2}=V_{SATH}$ valid for $t_1 < t < t_2$

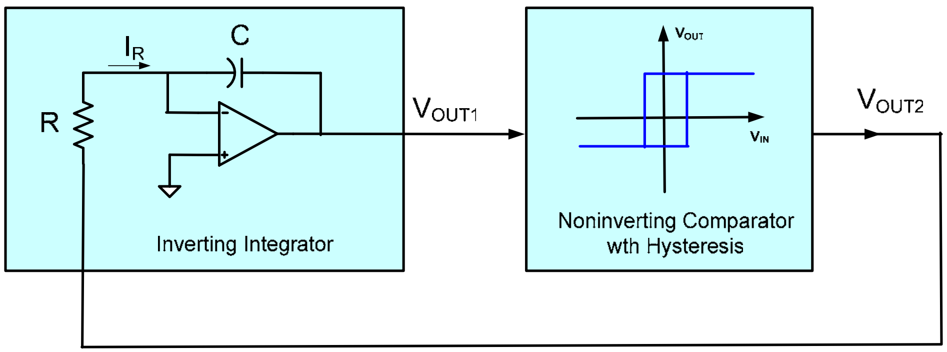
$$V_{HYL} = -\frac{1}{RC}\int_{t_1}^{t} V_{SATH}\,d\tau + V_{HYH}$$

$$V_{HYL} = -\frac{1}{RC}V_{SATH}\int_{t_1}^{t_2} 1\,d\tau + V_{HYH}$$

$$V_{HYL} = -\frac{1}{RC}V_{SATH}\left(\tau\Big|_{t_1}^{t_2}\right) + V_{HYH}$$

$$V_{HYL} = -\frac{1}{RC}V_{SATH}\left(t_2 - t_1\right) + V_{HYH}$$

Waveform Generator with Linear Triangle Waveform



$$V_{SATH} \cong V_{DD}$$

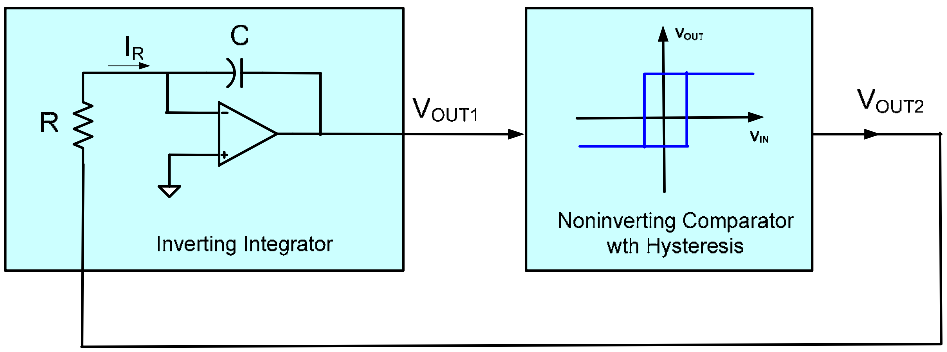
$$V_{SATL} \cong V_{SS}$$

Guess $V_{OUT2}=V_{SATH}$ valid for $t_1 < t < t_2$

$$V_{HYL} = -\frac{1}{RC} V_{SATH}\left(t_2 - t_1\right) + V_{HYH}$$

$$t_2 - t_1 = RC\frac{\left(V_{HYH} - V_{HYL}\right)}{V_{SATH}}$$

Waveform Generator with Linear Triangle Waveform



$$V_{SATH} \cong V_{DD}$$

$$V_{SATL} \cong V_{SS}$$

Guess $V_{OUT2}=V_{SATL}$ will obtain t_3-t_2 valid for $t_2 < t < t_3$

Following the same approach observe

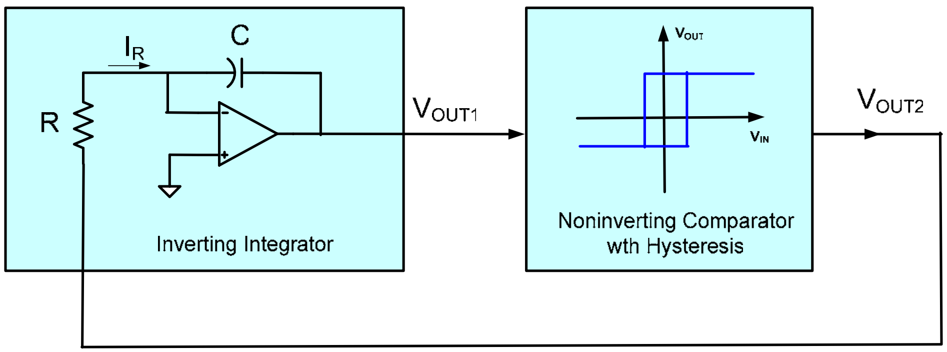
$$V_{OUT1} = -\frac{1}{RC}\int_{t_2}^{t} V_{SATL}\,d\tau + V_{OUT1}(t_2)$$

$$V_{OUT1}(t_2) = V_{HYL}$$

It thus follows that

$$V_{HYH} = -\frac{1}{RC} V_{SATL}(t_3 - t_2) + V_{HYL} \qquad t_3 - t_2 = RC\frac{(V_{HYL} - V_{HYH})}{V_{SATL}}$$

Waveform Generator with Linear Triangle Waveform



$$V_{SATH} \cong V_{DD}$$

$$V_{SATL} \cong V_{SS}$$

$$T = (t_2 - t_1) + (t_3 - t_2)$$

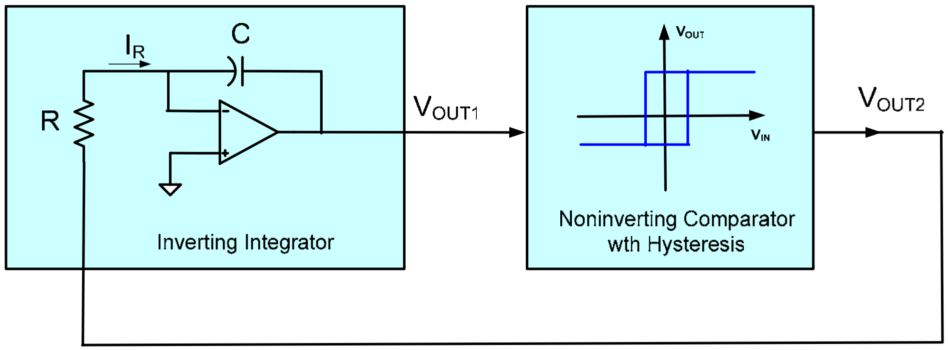
$$t_2 - t_1 = RC\frac{(V_{HYH} - V_{HYL})}{V_{SATH}}$$

$$t_3 - t_2 = RC\frac{(V_{HYL} - V_{HYH})}{V_{SATL}}$$

$$T = RC(V_{HYH} - V_{HYL})\left(\frac{1}{V_{SATH}} - \frac{1}{V_{SATL}}\right)$$

$$f = \frac{1}{t} = \frac{1}{RC}\frac{V_{SATL}V_{SATH}}{(V_{HYH} - V_{HYL})(V_{SATL} - V_{SATH})}$$

Waveform Generator with Linear Triangle Waveform



$$f=\frac{1}{RC}\frac{V_{SATL}V_{SATH}}{(V_{HYH}-V_{HYL})(V_{SATL}-V_{SATH})}$$

If we use the noninverting comparator with hysteresis circuit developed previously and if

If $V_{SATH}=V_{DD}$, $V_{STAL}=V_{SS}=-V_{DD}$ $\theta=\frac{R_1}{R_1+R_2}$

then

$$V_{HYH}=\frac{\theta}{1-\theta}V_{DD} \qquad V_{HYL}=\frac{-\theta}{1-\theta}V_{DD}$$

$$f=\frac{1}{2RC}\frac{1-\theta}{\theta}$$